Rutgers University: Algebra Written Qualifying Exam
January 2017: Problem 3

Exercise. Prove there are no simple groups of order 18.

Let G be a group of order 18. We want to show that there is a normal subgroup of G that is **not** $\{e\}$ or G. So first find the prime factors of $|G| = 80$.

$$18 = 2 \cdot 3^2.$$

By the third Sylow Theorem,

$$n_3 \equiv 1 \mod 3 \qquad \text{and} \qquad n_3 \mid 2$$
$$\implies n_3 = 1$$

Thus the number of $3-$Sylow subgroups is $n_3 = 1$.
Therefore, the $3-$Sylow subgroup is normal by the Second Sylow Theorem.
Thus, G is not simple.